IAN INVESTIGATION OF COMPRESSIBLE FLOWS WITH LARGE WHIRL COMPONENTS

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AN INVESTIGATION OF COMPRESSIBLE FLOWS
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by

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## ABSTRACT

This investigation was conducted to determine the losses in the scroll and inlet guide vanes of a dual discharge, radial-inflow turbine. Difficulties are encountered in such tests because the air is discharged from the guide vanes with a large whirl component into the cavity normally occupied by the turbine rotor. Connected with such flows are phenomena such as choking and energy separation. This led to the investigation of the flow in a vortex chamber where the vortex is driven by the inlet guide vanes of the radial turbine.

The air tests were conducted at the Turbo-Propulsion Laboratory of the Naval Postgraduate School, Monterey, California.

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| Actual | Definition | FORTRAN | Units |
| :---: | :---: | :---: | :---: |
| $A_{1}$ | Meridional crosssectional area at dummy rotor inlet | A1 | $f t^{2}$ |
| ${ }^{A} \delta$ | Cross-sectional area of scroll without friction |  | $i n^{2}$ |
| ${ }^{A_{\delta f}}$ | Cross-sectional area of scroll corrected for friction |  | $i n^{2}$ |
| $\mathrm{A}_{5}$ | Cross-sectional area of five-inch pipe |  | $f t^{2}$ |
| a | Acoustic velocity |  | $f t / s e c$ |
| $a_{i}$ | Acoustic velocity at radius $\mathrm{R}_{i}$ |  | $f t / s e c$ |
| $\mathrm{B}_{1}$ | Distance between turbine rotor shrouds at dummy rotor inlet | B1 | in |
| b | Width of vortex chamber |  | in |
| C | Factor dependent on orifice diameter and type of pressure taps used |  |  |
| $\mathrm{C}_{\mathrm{f}}$ | Conversion factor | CF1 | $\begin{aligned} & \text { lb/ft } t^{2} / \\ & \text { in } \mathrm{Hg} \end{aligned}$ |
| c | Constant defined in Appendix B |  | $f t$ |
| ${ }^{c} p$ | Specific heat at constant pressure | CP | $\begin{aligned} & \mathrm{BTU} / \mathrm{O}_{\mathrm{R}} \\ & \mathrm{Ib} \end{aligned}$ |
| F | Scale reading | SR | 1 b |
| $\mathrm{F}_{\mathrm{r}}$ | Reynolds number correction factor |  |  |
| $f$ | Skin friction coefficient |  |  |
| $\mathrm{G}_{\mathrm{Hg}}$ | Specific Gravity of mercury at $t_{r m}$ | GHG |  |


| Actual | Definition | FORTRAN | Units |
| :---: | :---: | :---: | :---: |
| $\mathrm{G}_{\mathrm{H}_{2} \mathrm{O}}$ | Specific gravity of water at $t_{r m}$ | GWR |  |
| g | Gravitational constant | 32.174 | $\begin{aligned} & l b_{m}-f t / \\ & l b-\sec ^{2} \end{aligned}$ |
| hatm | Measured atmospheric pressure | HATM | in $\mathrm{H}_{2} \mathrm{O}$ |
| $\mathrm{h}_{1}$ | Average measured pressure at dummy rotor inlet | H1 | in $\mathrm{H}_{2} \mathrm{O}$ |
| h | Measured pressure as appropriate |  | in $\mathrm{H}_{2} \mathrm{O}$ |
| $\mathrm{h}_{\text {ref }}$ | Measured static pressure in five-inch pipe |  | in $\mathrm{H}_{2} \mathrm{O}$ |
| $h_{1 v c}{ }^{\prime}$ | Measured pressure across orifice (vena contracta taps) | DPVC | cm Hg |
| $\triangle h_{\text {IVc }}$ | Actual pressure across orifice (vena contracta taps) | DVC |  |
| J | Conversion factor | 778.16 | $f t-1 b / B t u$ |
| M | Mach number |  |  |
| M | Moment exerted on dummy rotor | M | $f t-1 b$ |
| $M_{i}$ | Mach number at radius $\mathrm{R}_{i}$ |  |  |
| $M_{r}$ | Ratio of measured dynamic pressure to measured absolute total pressure for probe calibration |  |  |
| $M_{1}$ | Mach number at dummy rotor inlet | ACH1 |  |
| $\mathrm{MV}_{4}$ | Thermocouple reading ahead of orifice | V4 | mv |
| $\mathrm{MV}_{5}$ | Thermocouple reading at manifold inlet | V5 | mv |
| $\mathrm{P}_{\text {atm }}$ | Atmospheric pressure (barometer) | PAT | in Hg |


| Actual | Definition | FORTRAN | Units |
| :---: | :---: | :---: | :---: |
| $\mathrm{P}_{\mathrm{t}}$ | Total pressure as appropriate |  |  |
| $\mathrm{P}_{\text {to }}$ | Total pressure at manifold inlet | PTO | in Hg abs |
| $P_{\text {t2 }}$ | Total pressure at rotor discharge section |  | in Hg abs |
| p | Absolute static pressure as appropriate |  |  |
| $p_{i}$ | Absolute static pressure at radius $\mathrm{R}_{\mathrm{i}}$ |  |  |
| $p_{0}$ | Absolute static pressure at manifold inlet | PS5 | cm Hg abs |
| $\mathrm{p}_{1}$ | Average static pressure at dummy rotor inlet | P1 | in Hg |
| $\mathrm{p}_{2}$ | Actual static pressure at rotor discharge section |  | $\mathrm{cm} \mathrm{H}_{2} \mathrm{O}$ |
| $\mathrm{p}_{5}{ }^{\prime}$ | Measured static pressure at manifold inlet | P5P | cm Hg gage |
| $\mathrm{p}_{1 \mathrm{Vc}}{ }^{\prime}$ | Measured pressure upstream of orifice (vena contracta taps) | PUVC | cm Hg gage |
| $\mathrm{p}_{1 \mathrm{Vc}}$ | Absolute pressure ahead of orifice (vena contracta taps) | PVC | cm Hg abs |
| $\mathrm{P}_{\text {to }} / \mathrm{p}_{1}$ | Ratio of total pressure at manifold inlet to static pressure at dummy rotor inlet | PR |  |
| $\mathrm{P}_{1}-p_{2}$ | Measured dynamic pressure |  | cm $\mathrm{H}_{2} \mathrm{O}$ |
| $p_{4}-p_{5}$ | Measured pressure difference between the two pitch angle pressure taps |  | cm $\mathrm{H}_{2} \mathrm{O}$ |
| $\frac{P_{1}-P_{t}}{P_{t}-P_{s}}$ | Total pressure coefficient |  |  |


| Actual | Definition | FORTRAN | Units |
| :---: | :---: | :---: | :---: |
| $\frac{\mathrm{p}_{4}-\mathrm{p}_{5}}{\mathrm{p}_{1}-\mathrm{p}_{2}}$ | Pitch angle pressure coefficient |  |  |
| $\frac{\mathrm{P}_{\mathrm{t}}-\mathrm{p}_{\mathrm{s}}}{\mathrm{P}_{1}-\mathrm{p}_{2}}$ | Velocity pressure coefficient |  |  |
| $\mathrm{P}_{\mathrm{t}}-\mathrm{p}_{\mathrm{s}}$ | Actual dynamic pressure |  | $\mathrm{cm} \mathrm{H}_{2} \mathrm{O}$ |
| R | Radius as appropriate |  |  |
| $\mathrm{R}_{1}$ | Radius at dummy rotor inlet |  | ft |
| $\mathrm{R}_{\mathrm{i}}$ | Injection radius where conditions $M_{i}$ and $\alpha_{i}$ are known |  | ft |
| $\mathrm{R}_{0}$ | Rotor shroud radius at rotor discharge section |  | in |
| $r$ | Radius of scroll crosssectional area |  | $f t$ |
| $r_{1}$ | Inside radius of scroll |  | $f t$ |
| $r_{f}$ | Radius of scroll crosssectional area corrected for friction |  | ft |
| T | Static temperature |  | ${ }^{\circ} \mathrm{R}$ |
| $\mathrm{T}_{\mathrm{i}}$ | Static temperature at radius $R_{i}$ |  | ${ }^{\circ} \mathrm{R}$ |
| $\mathrm{T}_{\mathrm{t}}$ | Total temperature as appropriate |  | ${ }^{\circ} \mathrm{R}$ |
| $\mathrm{T}_{\text {to }}$ | Total temperature at manifold inlet | T5 | ${ }^{0} \mathrm{R}$ |
| To | Static temperature at manifold inlet | TO | ${ }^{\circ} \mathrm{R}$ |
| $\mathrm{T}_{\mathrm{t} 2}$ | Total temperature at rotor discharge section |  | ${ }^{0} \mathrm{R}$ |
| $\mathrm{T}_{1}$ | Static temperature at dummy rotor inlet | T1 | ${ }^{\circ} \mathrm{R}$ |
| T4 | Temperature ahead of orifice | T4 | ${ }^{\circ} \mathrm{R}$ |


| Actual | Definition | FORTRAN | Units |
| :---: | :---: | :---: | :---: |
| t | Temperature as appropriate | A | ${ }^{\circ} \mathrm{F}$ |
| tare | Tare of mercury micromanometer | TARE | cm Hg |
| tare | Tare of precision scale | STARE | 1 b |
| $t_{c j}$ | Cold junction temperature | TCJ | ${ }^{\circ} \mathrm{F}$ |
| $t_{r m}$ | Control room temperature | TRM | ${ }^{\circ} \mathrm{F}$ |
| V | Velocity |  | $\mathrm{ft} / \mathrm{sec}$ |
| $\mathrm{V}_{\mathrm{i}}$ | Velocity at radius $\mathrm{R}_{\mathrm{i}}$ |  | $f t / s e c$ |
| $\mathrm{V}_{\mathrm{m}}$ | Meridional velocity component |  | $\mathrm{ft} / \mathrm{sec}$ |
| $\mathrm{V}_{0}$ | Velocity at manifold inlet | V1 10 ). | $f t / s e c$ |
| $\mathrm{V}_{\mathrm{mi}}$ | Meridional velocity component at radius $\mathrm{R}_{\mathrm{i}}$ |  | $f t / s e c$ |
| $\mathrm{V}_{\mathrm{u}}$ | Peripheral velocity component |  | $f t / s e c$ |
| $\mathrm{V}_{\text {ui }}$ | Peripheral velocity component at radius $\mathrm{R}_{\mathrm{i}}$ |  | $\mathrm{ft} / \mathrm{sec}$ |
| $\mathrm{V}_{1}$ | Velocity at dummy rotor inlet | V1 | $\mathrm{ft} / \mathrm{sec}$ |
| $\mathrm{V}_{1}$ th | Theoretical velocity at dummy rotor inlet |  | $\mathrm{ft} / \mathrm{sec}$ |
| $\mathrm{V}_{\mathrm{m} 1}$ | Meridional component of $\mathrm{V}_{1}$ | VM1 | $\mathrm{ft} / \mathrm{sec}$ |
| $\mathrm{V}_{\text {u1 }}$ | Peripheral component of $\mathrm{V}_{1}$ | VU1 | $\mathrm{ft} / \mathrm{sec}$ |
| $\mathrm{V}_{2}$ | Velocity at rotor discharge section |  | $\mathrm{ft} / \mathrm{sec}$ |
| ${ }^{\mathrm{V}}$ ¢ 1 | Tangential velocity component at inside radius of scroll |  | $\mathrm{ft} / \mathrm{sec}$ |
| V | Volume flow rate |  | $\mathrm{ft}^{3} / \mathrm{sec}$ |
| $\dot{W}_{\text {VC }}$ | Mass flow rate (vena contracta taps) | WVC | $1 \mathrm{~b}_{\mathrm{m}} / \mathrm{sec}$ |
| X | Reynolds number factor | X |  |


| Actual | Definition | FORTRAN | Units |
| :---: | :---: | :---: | :---: |
| $Y_{1}$ | Expansion factor accounting for thermal expansion of orifice | Y |  |
| Z | Absolute viscosity |  | centipoises |
| \% | Axial coordinate in vortex chamber measured from center plane | z | in |
| $\alpha$ | Area multiplier accounting for thermal expansion of orifice | A |  |
| $\alpha$ | Flow angle |  | degrees |
| $\alpha_{i}$ | Flow angle at radius $\mathrm{R}_{1}$ |  | degrees |
| $\alpha_{1}$ | Absolute dummy rotor inlet flow angle | ALP1 | degrees |
| $\alpha_{2}$ | Measured yaw angle of flow at rotor discharge section |  | degrees |
| $\gamma$ | Ratio of specific heats | GAM |  |
| $\frac{\gamma-1}{\gamma}$ | Exponent using $\gamma$ | EXP |  |
| $\delta$ | Circumferential angle of advance of scroll measured from inlet pipe centerline |  | degrees |
| $\Delta q$ | Nondimensional pressure loss in scroll |  |  |
| $\varnothing$ | Velocity coefficient of scroll and guide vanes | PHI |  |
| 0 | Density |  | $1 b_{m} / f^{3}$ |
| $O_{i}$ | Density at radius $\mathrm{R}_{1}$ |  | $1 b_{m} / \mathrm{ft}^{3}$ |
| $\rho_{0}$ | Density at manifold inlet | RHO | $1 b_{m} / f^{3}$ |
| $\rho_{1}$ | Density at dummy rotor rotor. inlet | RHO | $1 b_{m} / \mathrm{ft}^{3}$ |
| $\theta_{2}$ | Measured pitch angle of flow at rotor discharge section |  | degrees |

1. Introduction

For axial turbine stages it is possible to determine, at least in a fairly accurate manner, the losses in guide vanes by passing air through the axial cascade with the rotor removed. Nevertheless, errors occur if blade heights are large with respect to the blading diameter. These errors are due to the fact that the rotor blading will set up radial pressure gradients after the guide vanes so that the guide vanes will not discharge into a constant pressure region as they do in tests with rotor removed.

If the guide vanes of a radial turbine are tested with the rotor removed, conditions exist which make direct measurements of the required parameters difficult. Large whirl components of velocity are produced after the inlet guide vanes in the cavity which would normally be occupied by the turbine rotor. These whirl components increase with decreasing radius if angular momentum is conserved. This whirling flow may lead to such phenomena as choking and energy separation. Flow conditions may then be completely different in the vaneless space after the guide vanes than with the turbine rotor installed. Vavra $[1]^{1}$ has surveyed the discharge of the inlet guide vanes of a radial turbine and determined that it was possible to measure a higher total pressure after the guide vanes than was known to exist before them. This seems to be in violation of the principle of the conservation of energy. It was also determined that a

1 Numbers in brackets refer to bibliography entries of section 8 .
considerable vacuum core existed near the centerline causing ambient air to flow in and mix with the discharge air of the guide vanes. It seemed possible that this effect and the interesting but not well understood phenomenon known as the "Ranque-Hilsch effect" might be connected with the experienced conditions.

The subject investigation was undertaken to determine losses in the scroll and inlet guide vanes of a radial turbine and to better understand the conditions existing in the flow after the guide vanes with the turbine rotor removed.

Riley [2] has investigated the effect of axial clearance on the performance of a radial turbine and found that the efficiency of the turbine remained relatively constant until a "critical" clearance was reached beyond which point a further increase in clearance caused the efficiency to decrease rapidly. In determining some performance parameters it was assumed that the velocity coefficient, which is a measure of the losses occurring in the inlet manifold and guide vanes, as well as the absolute rotor inlet angle, were constant in the range of turbine pressure ratio and axial clearance tested. It was desired to conduct more extensive tests of the inlet manifold and guide vanes so that a more definitive analysis of Riley's data could be made. If the investigation led to the possibility of improvements in the existing installation, these would be incorporated before future turbine tests were conducted.

Essentially four separate but related investigations were carried out. Conditions at the rotor inlet and losses in the inlet manifold and guide vanes were evaluated from dummy rotor torque tests. A computer program was used to reduce data and calculate the desired parameters from these tests. Surveys were conducted to determine the flow conditions in the plane of the turbine rotor discharge with the rotor removed. Pressure measurements were made around the periphery of the radial turbine inlet manifold to ascertain whether the scroll could be improved. The turbine rotor shrouds were replaced with flat plates which formed vortex chamber sides, and an investigation of the flow in the vortex chamber was conducted.

Whirling flows have increased in importance not only in the field of radial or mixed-flow turbines and compressors, but also in studies relating to spin stabilized solid propellant rockets, particle separators, and gaseous nuclear rockets.

The author gratefully acknowledges the assistance and encouragement given by Dr. M. H. Vavra of the Department of Aeronautics.
2. General Installation and Instrumentation

The same basic installation was used in all the subject investigations, with some changes in components and instrumentation for the different tests. It consisted of the scroll type inlet manifold and the inlet guide vanes of a dual discharge, radial-inflow turbine. The unit is located at the Turbo-Propulsion Laboratory of the Department of Aeronautics at the Naval Postgraduate School. The scroll type inlet manifold is of laminated wood with the inner contour formed by casting plaster of Paris around a wooden insert. This insert which is in the shape of a scroll or torus of varying cross-sectional area is shown in Fig. 1. The manifold casing which was constructed in halves has been sanded and varnished internally to provide a smooth flow surface and to prevent erosion. The seven inlet guide vanes which have circular arc profiles are held between two rings, which are in turn fastened to the manifold casing when it is assembled. Fig. 2 is a cross section of the manifold casing showing the scroll shape and inlet guide vane positions.

The source of air for all tests is an Allis-Chalmers 12-stage axial flow compressor which provides high pressure air for the three test cells at the Turbo-Propulsion Laboratory. Air from the main supply line passes through a fourinch pipe with a flow-measuring orifice, a settling tank, and a five-inch pipe with flow straighteners, before entering the turbine inlet manifold. The flow rate is regulated from the control room by two remotely controlled butterfly valves which are shown in Fig. 3.

A sharp-edged orifice, which conforms to standards set forth by Stearns, et al. [3], is located in the four-inch pipe which is shown in Fig. 3. The pressure ahead of the orifice and the pressure difference across the orifice were obtained with standard flange and vena contracta taps. A chromel-alumel thermocouple installed in a Kiel temperature probe was used to measure the temperature ahead of the orifice.

Thermocouple voltage potentials were measured on a 48 channel Brown Potentiometer manufactured by Minneapolis Honeywell. The potentiometer shown in Fig. 4 is located in the control room. The reference or cold junction temperature is maintained at $32^{\circ} \mathrm{F}$ by an ice bath in the test cell.

The pressures obtained from the vena contracta and flange taps of the flow-measuring orifice, and the wall static pressure in the five-inch pipe at the manifold inlet, were measured with an accuracy of $\pm 0.01 \mathrm{~cm}$. on a 100 cm . mercury micromanometer in the control room. The wall static pressure in the five-inch pipe at the manifold inlet can also be read on a Heise pressure gage in the control room. The Heise gage is shown in Fig. 4. This gage was usually used to set the pressure ratio for test runs. When it was desired to set the same conditions in different test runs as nearly as possible, the mercury micromanometer was used to reset the wall static pressure in the five-inch pipe. The total pressure and total temperature were sensed at the manifold inlet at the centerline of the five-inch pipe with a Kiel probe containing a chromel-alumel thermocouple.

A 96-inch water manometer board in the control room was used to measure static pressures around the scroll periphery, the static pressures ahead of the inlet guide vanes, and the static pressures after the inlet guide vanes or in the vortex chamber side walls depending on the test. The pressure heads were measured with an accuracy of $\pm 0.05$ inches. Atmospheric and/or wall static pressures in the five-inch pipe were used as reference pressures for the reservoirs of the 96-inch manometer board.

Equipment and instrumentation not utilized in all tests are described separately in the sections concerning the test to which they are applicable.
3. Scroll and Guide Vane Performance from Torque Measurements

## Installation

Because difficulties were encountered in attempting to evaluate the inlet guide vane discharge conditions from pressure surveys, Vavra [1] and subsequently Riley [2] used a special installation to determine the losses in the scroll and guide vanes as well as the absolute flow angle at the rotor inlet. This installation actually determines average conditions at a position which would normally be the rotor inlet radius, thereby including changes in the flow conditions in the vaneless space between the inlet guide vanes and the rotor. This information may be more useful in the determination of performance parameters. It was desired to make improvements to the installation and to conduct more extensive tests especially to investigate the effect of increased axial clearance on this method of performance determination.

A dummy rotor with 36 meridional blades that approximate the contour of the turbine rotor shrouds is supported on self-aligning ball bearings and replaces the turbine rotor. The dummy rotor turns the flow that is discharged by the guide vanes to the axial direction at the rotor discharge. If the flow leaves the rotor with no peripheral velocity component, it is possible to determine the losses in the guide vanes by measuring the torque exerted on the static dummy rotor, the flow rate, the conditions at the manifold inlet, and the static pressure at the dummy
rotor inlet. The dummy rotor shown in Fig. 5 has a diameter of 9.50 inches. This places the blade leading edges at the radius which corresponds with the radius of the annulus in which the static pressure taps ahead of the rotor are located, and at a 0.050 inch greater radius than the actual rotor inlet. The pressure taps depicted in the cross section in Fig. 6 are placed at eight equidistant positions around the periphery of the so-called right-hand side and left-hand side plexiglas rotor shrouds. A total of sixteen pressure taps are individually connected to the 96 -inch water manometer board. The axial length of the dummy rotor is 8.50 inches, which extends the blade trailing edges beyond the shrouds shown in Fig. 6.

To insure that the discharge flow was without whirl components, new flow straighteners were fabricated and attached to the dummy rotor for these tests. The flow straighteners shown in Fig. 5 are aluminum caps which slide over the rotor shaft and cover the rotor blade trailing edges. The caps contain a 1.00 inch depth of 0.125 by 0.0015 inch honeycomb material to insure an axial discharge.

The entire dummy rotor assembly is supported by brackets attached to the manifold casing as shown in Fig. 7 . The rotor was centered in the shrouds with shims so that there was a uniform radial clearance at the discharge of 0.035 inches. The axial clearance was varied in individual test runs by placing circular shims or gaskets between the shrouds and the manifold casing. A twelve-inch lever arm
was attached to the rotor shaft so that the moment exerted on the dummy rotor by the flow could be measured with a 25 pound capacity Toledo precision scale. The lever arm is attached horizontally and the force is exerted on the scale through an exactly vertical link so that scale readings and moment are directly related. To extend the capacity of the scale and to allow tests at higher pressure ratios (and mass flow rates) a counterbalance was added to the lever arm. The installation for these tests is shown in Fig. 7.

## Analytical Derivation and Data Reduction

The integrated pressures acting over the inlet and discharge areas of the dummy rotor cannot produce a moment about the axis. The flow straighteners insure that the discharge velocity is axial. Then from the law of moment of momentum $^{1}$, for a steady flow that does not have a peripheral velocity component at the discharge section, the moment $M$ exerted on the dummy rotor is

$$
\begin{equation*}
M=\frac{\dot{W}_{v c}}{g} R_{1} V_{u_{1}} \tag{3-1}
\end{equation*}
$$

where $\dot{W}_{V c}$ is the mass flow rate determined by using the vena contracta taps of the flow measuring orifice. $R_{1}$ is the radius at the inlet of the dummy rotor and $V_{u 1}$ is the peripheral velocity component at $R_{1}$. The velocity coefficient $\phi$, which is a measure of the performance of the
${ }^{1}$ Vavra, M. H. Aero-Thermodynamics and Flow in Turbomachines (John Wiley and Sons, 1960), p. 98.
scroll and inlet guide vanes, is defined by

$$
\begin{equation*}
\phi \equiv \frac{V_{I}}{V_{I t h}} \tag{3-2}
\end{equation*}
$$

where $V_{1}$ is the actual average velocity at the dummy rotor inlet section and $V_{1 \text { th }}$ is the theoretical velocity for an isentropic expansion from the manifold inlet conditions $P_{\text {to }}, T_{\text {to }}$ to the static pressure $p_{1}$ at the dummy rotor inlet. From the energy equation for an adiabatic process, the velocity $V_{1}$ at the dummy rotor inlet is

$$
\begin{equation*}
V_{1}=\sqrt{2 g J_{c_{p}}\left(T_{t o}-T_{1}\right)} . \tag{3-3}
\end{equation*}
$$

or using the isentropic relation for a perfect gas and equation (3-3),

$$
\begin{equation*}
V_{1}=\phi \sqrt{2 g J_{c p} T_{t_{0}}\left[1-\left(\frac{p_{1}}{P_{t_{0}}}\right)^{\frac{\gamma-1}{\gamma}}\right]} \tag{3-4}
\end{equation*}
$$

From the equation of continuity, expressed at the dummy notor inlet, the average meridional component of velocity $\mathrm{V}_{\mathrm{m} 1}$ is

$$
\begin{equation*}
V_{m 1}=\frac{\dot{W}_{v c}}{\rho_{1} A_{1}} \tag{3-5}
\end{equation*}
$$

where $A_{1}$ is the meridional cross section and $\rho_{1}$ the density at the dummy rotor inlet. The area, $A_{1}$, is

$$
A_{1}=2 \pi R_{1} B_{1}
$$

where $B_{1}$ is the distance between the shrouds at the dummy rotor inlet which varies with the set axial clearance. From the equation of state for a perfect gas, the density $\rho_{1}$, is

$$
\begin{equation*}
\rho_{1}=\frac{P_{1}}{R_{g} T_{1}} \tag{3-7}
\end{equation*}
$$

Then the meridional velocity $\mathrm{V}_{\mathrm{m} 1}$ is

The velocity $V_{1}$ is

$$
\begin{align*}
& V_{m 1}=\frac{\dot{W}_{v c} R_{g} T_{1}}{A_{1} p_{1}}  \tag{3-8}\\
& V_{1}=\sqrt{V_{m 1}^{2}+V_{u 1}^{2}} \tag{3-9}
\end{align*}
$$

Equations (3-3), (3-4) and (3-9) are expressions for the velocity $V_{1}$, which after the substitution of equations (3-1) and (3-8) into equation (3-1) will have three unknowns. The unknown quantities to be determined are the velocity $V_{1}$, the velocity coefficient $\boldsymbol{\phi}$, and the temperature $\mathrm{T}_{1}$. All other quantities are either constants or quantities determined directly from measured data. The computer program SCROLL which is described in detail and listed in Appendix A determines the unknowns from test data using an iteration process. With the velocity components determined, the absolute flow angle at the dummy rotor inlet is

$$
\begin{equation*}
\alpha_{1}=\tan ^{-1} \frac{V_{u 1}}{V_{m 1}} \tag{3-10}
\end{equation*}
$$

Program SCROLI makes temperature corrections to the physical properties of air and the specific gravity of water and mercury used in the respective manometers. It converts manometer readings and thermocouple outputs into the required pressures and temperatures, respectively. The program then establishes the mass flow rate and calculates the average conditions at the dummy rotor inlet from the torque measurement. The program was processed on the Control Data Corporation 1604 Computer at the Naval Postgraduate School.

## Testing and Results

A total of five test runs were completed. Program SCROLL input and output data for these runs are tabulated in Appendix A. The velocity coefficient and flow angle as functions of pressure ratio are shown graphically in Fig. 8, 9 and 10. Data for a single test run made by Riley [2] are also shown.

The initial test run was conducted at minimum axial clearance but with the flow straighteners installed. Because the pressures $p_{1}$ at the dummy rotor inlet are sensed at sixteen positions, Polaroid photographs were used to record the manometer board. The pressure heads were averaged for use in program SCROLL. Velocity coefficients near the design point pressure ratio of the turbine are approximately two percentage points higher for this first run than those determined by Riley [2]. This seemed to indicate that the addition of flow straighteners eliminated any peripheral velocity components at the discharge which may have existed with the former installation.

For the second run circular aluminum shims of 0.030 inch thickness were placed between the shrouds and the manifold casing. With this increased axial clearance, the resulting velocity coefficients were about one percentage point lower than with the minimum clearance in the range of interest.

During the first two test runs it was observed that the static pressures measured at the dummy rotor inlet
varied considerably from position to position around the periphery of the shroud. They were also observed to be sensitive to the relative positions of the rotor blade leading edges. To reduce the effects of local perturbations, the pressure tap positions were moved axially outward in the annulus around the periphery of the shroud. To prevent leakage between the shrouds and manifold casing 0.015 inch thick circular gaskets were placed there for the third run. This effectively increased the axial clearance by the thickness of the gasket. The repositioning of the pressure taps greatly improved the uniformity of pressures sensed around the shrouds.

The velocity coefficients resulting from the third run were larger than in the preceding runs. This may indicate that the static pressures recorded were nearer to actual average conditions existing at the rotor inlet. The pressure ratio $P_{\text {to }} / p_{1}$ was thereby reduced and the theoretical velocity $V_{1 t h}$ decreased correspondingly, thus increasing the resulting velocity coefficients $\phi$ for the same flow conditions.

It was noted in the first three runs that pressures sensed on the right side were somewhat greater than those on the left. Measurements showed that the inside extremity of the left shroud was offset 0.016 inch more from the guide vane ring than the right shroud. The dummy rotor was centered between the shrouds for all runs. To improve the symmetry of conditions in the final runs, a greater
thickness of gasket material was placed between the right shroud and the manifold casing.

In an attempt to eliminate influences other than dummy rotor axial tip clearance, run No. 5 was conducted immediately following run No. 4. The only change to the installation between these runs was the reduction of the axial clearance by 0.025 inch by the removal of a 0.025 inch thick gasket from under each shroud. The clearances were 0.072 and 0.047 inch for runs 4 and 5 respectively. The data from run 5 compares very well with that from run 3 , which was conducted at a 0.051 inch clearance. The velocity coefficient near design pressure ratio determined by runs 3 and 5 is approximately 92.5 percent. The clearances set for runs 1 and 2 were 0.036 and 0.066 inch, respectively. A comparison of runs 1 and 2 and of runs 4 and 5 indicates that an increase in axial clearance, of the amounts made, decreases the velocity coefficient by approximately one percent. An explanation of this may be that the angular momentum of the flow escaping around the blade tips is decreased by wall friction before it impinges on the dummy rotor blades at a lesser radius. It appears then that the most accurate performance would be obtained from the type of test conducted where the blade profile conforms closely to the shroud contour and where the axial clearance is minimized near the dummy rotor blade tips.

Riley [2] used a distance of 0.943 inch between the shrouds at the rotor inlet for his analysis. Since the
axial width of the blade tips of the dummy rotor is 0.894 inch, this would indicate that there existed an axial clearance of 0.0245 inch. The axial clearances in the subject tests were larger than in Riley's test because the distance between shrouds was determined to be 0.966 inch without gaskets or shims installed. The axial clearance set in the different tests ranged from a minimum of 0.036 inch to 0.072 : inch. Since the manifold casing had been disassembled and varnished internally after Riley's test, the distance between the shrouds probably changed. The wooden manifold casing is also subject to changes in dimensions with changes in atmospheric humidity.

The axial distance between the shrouds has a limited effect on the calculation of the velocity coefficient; however, since it is used directly in calculation of the meridional velocity component from the mass flow rate, it influences the rotor inlet angle $\alpha_{1}$ considerably. The decrease in $\alpha_{1}$ in runs 1 and 5 in Fig. 10 is in the main due to the increase in the calculated meridional velocity component for a decrease in the distance between shrouds. In any given run $\boldsymbol{\alpha}_{1}$ decreases with increasing pressure ratio due to the requirement that it decreases with increasing mass flow rate.

Fig. 8 and 9 indicate that the velocity coefficient increases slightly with increasing pressure ratio. If the internal surfaces are smooth enough, this result may be due to a decreasing coefficient of friction with increasing Reynolds number.

Conclusions and Recommendations
The velocity coefficient $\phi$ for the inlet manifold and guide vanes is at least $92.5 \%$. If the axial tip clearance could be reduced to a minimum while maintaining the rotational freedom of the dummy rotor, it is possible that an increased velocity coefficient would be determined indicating that the losses are in reality lower. This would require reworking the dummy rotor or turbine rotor shrouds.

Future testing of the radial turbine should be conducted using the static pressure taps at the rotor inlet in their new location in order to obtain more reliable data at this position.
4. Survey of Turbine Rotor Discharge Section with Rotor Removed

Because peculiar conditions such as extreme noise and high frequency vibrations exist if tests are carried out with the turbine rotor removed, this configuration was investigated to gain some insight into the existing flow conditions. The installation had been equipped to allow pressure and temperature surveys of the flow at the turbine rotor discharge in earlier turbine performance tests. Using the available instrumentation, the rotor discharge area was surveyed with the rotor removed. Because working at a higher pressure ratio was almost impossible due to the extreme noise and vibrations, a survey of the discharge area was conducted at $\frac{\mathrm{P}_{\text {to }}}{\mathrm{P}_{\text {atm }}}=1.25$ only. The flow was assumed to be axisymmetric and the pressure and temperature surveys were conducted at one peripheral position, but 90 degrees apart. The axial position of the survey plane at the discharge is shown in Fig. 6.

A United Sensor Model DA-120 pressure probe was used to survey the discharge section. This instrument is a threedimensional pressure probe capable of measuring the yaw and pitch angles, and total and static pressures. The probe holder was mounted on the upper surface of the inlet casing, and the probe passes through the casing and the shroud as shown in Fig. 6. The probe can be moved vertically and rotated about the vertical axis. Yaw angles are determined by rotating the probe to balance static pressures sensed on
either side of the wedge of the probe. With the measured pressure differential sensed by the axially spaced taps, 4 and 5, shown in Fig. 11, the pitch angle is determined from calibration curves for the probe. All pressures sensed by the probe were measured differentially on the $U$-tube water manometer board shown in Fig. 21. The probe is connected to the manometer board as shown in Fig. 12.

The total temperature probe used to survey the discharge section is depicted in Fig. 11. This probe, which utilizes an iron-constantant thermocouple, was manufactured locally. It was mounted on the end of the manifold casing in a holder similar to that used for the pressure probe. The temperature probe is located in the same axial plane but at a pe: ripheral position $90^{\circ}$ away from the pressure probe. The temperature probe was set at the same radial positions and yaw angles that were obtained by the pressure probe. The thermocouple output was measured on the potentiometer in the control room.

The results of the pressure and temperature surveys are shown graphically in Fig. 13 through 16. From the pressure distribution at the section surveyed it is apparent that the discharge flow is confined to the area where the radius ratio is between 0.75 and 1.00 . This area is approximately $43 \%$ of the total cross-sectional area. The central core was at a pressure below the atmospheric pressure with velocities either nonexistent or of magnitudes too small to establish reliable differential pressure information. The largest
velocities exist at the outer radius near the surface of the shroud. The velocity distribution was established by assuming isentropic recovery of total pressure by the probe. Since the total temperature was also measured, there is from the energy equation for an adiabatic flow,

$$
\begin{equation*}
V_{2}=\sqrt{2 g J c_{p} T_{t 2}\left[1-\left(\frac{P_{2}}{P_{t 2}}\right)^{\frac{\gamma-1}{\gamma}}\right]} \tag{2-1}
\end{equation*}
$$

The velocity distribution is shown in Fig. 16. The yaw angle shown in Fig. 14 is nearly constant in the range from $R / R_{0}=1.0$ to $R / R_{0}=0.9$, indicating that the axial and peripheral velocity profiles are similar. However at smaller radius ratios the flow turns considerably toward the axial direction.

The pitch angle distribution determined from the survey is shown in Fig. 14 also. The high pitch angles close to the shroud surface are considered to be in error. The determination of pitch angle is dependent upon the calibration curves which are obtained for deeply immersed probes. If measurements are made close to the wall the uppermost tap of the two pressure sensing taps is inside the probe access hole. Large pitch angles close to the wall are inconsistent with the physical situation. Because the static pressure is also dependent upon the calibration curves through the pitch angle, the calculated velocities shown in Fig. 16 may be too high near the shroud surface.

The largest velocities at a given pressure ratio can be expected to occur at the point of maximum curvature of
the meridional shroud contour before the flow is turned into the axial direction. From the survey it appears that all velocities are well below the acoustic velocity at the pressure ratio of the test. Nevertheless the noise and vibration level were extremely high and almost unbearable. The frequencies and amplitudes of the vibrations increased with increasing pressure ratio. Although the flow seemed to be stable from an overall point of view, the low pressure core seemed somewhat unstable and irregular. If smoke was released into the core from the outside it was sometimes carried up into the shrouds and sometimes it was ejected axially through the opposite side. Since the distance between shrouds increases rapidly with decreasing radius due to the curvature of the shrouds, the flow will decrease its radial velocity components considerably as it moves toward the axis. It may be possible that the flow midway between the shrouds approaches a solid body rotation without radial velocity components. A secondary flow condition may even exist with radial inward velocity components at the walls of the shrouds and radial outward velocity components midway between them. Such a rotating flow may extend radially outward to the inlet guide vanes and impose an unsteady condition as it passes the seven individual guide vanes or produce nonuniform pressure distributions in the scroll.

It might be of interest to determine the change of the frequencies of the vibrations with pressure ratio or flow rate, to verify whether a correlation exists between the frequency of vibration and the rotational velocity of the flow.
5. Scroll Performance

The turbine inlet manifold is described in section 3 . It has a complex, three-dimensional shape that makes an analytical study of the flow difficult. The shape of the scroll is depicted in Fig. 1 and 2. The purpose of the scroll is to produce a uniform flow with constant static and total pressure ahead of the inlet guide vanes. The average velocity would then be constant at the leading edges of the guide vanes everywhere on the periphery. With losses however both conditions cannot be realized and a uniform static pressure around the periphery may be the more important criteria. A constant pressure around the periphery prevents the rotor from experiencing varying forces and radial thrust. Unbalanced flow may accelerate the wear of bearings and seals. To accomplish the desired conditions the cross-sectional area of the scroll must decrease with the angle of advance. If the flow in the scroll is considered frictionless, angular momentum is conserved and the velocity increases with decreasing mean radius. This effect will be reduced somewhat by wall friction. The effects of friction and the displacement thickness of the boundary layer require that cross-sectional areas be increased over those for frictionless flow. Due to the complexity of the flow, the loss in angular momentum is usually neglected and the blockage due to wall friction is approximated by pipe data. Most analyses use information obtained with centrifugal pumps and compressor volutes, or it is frequently stated in the
literature that designers must account for these effects by experience. A nearly circular cross section is desirable since it passes the largest volume flow rate per surface area, thereby minimizing wall friction losses. In radial turbines nearly-circular scroll cross sections can be used in conjunction with inlet guide vanes since the flow is accelerating and can pass smoothly from the manifold to the guide vanes.

The existing area distribution of the scroll is shown in Fig. 17. There is nearly a linear decrease in area with angle of advance. Eckert [13] gives a relation that establishes the required flow area as a function of angular position for a compressor volute. This relation, although approximate, includes the effect of increased average velocity with decreased mean radius for circular cross sections. A second relation can be used to correct the determined areas for frictional losses. The relations can be applied to the turbine scroll if the terms are defined in an appropriate manner. An area distribution calculation is carried out in Appendix $B$ and the results are shown in Fig. 17. The area distribution resulting from the calculation is very close to that of the existing scroll.

In order to determine the pressure distribution in the present inlet manifold and to evaluate its performance, eleven static pressure taps were installed around the outer periphery. The positions of these taps, which were placed at 30 degree intervals near the bisecting plane of the scroll,
are shown in Fig. 2. Eight static pressure taps were already installed on either side of the scroll ahead of the inlet guide vanes. The positions of these taps are shown in Fig. 2 and 6. The pressures obtained by these static taps were measured on the 96 -inch water manometer board for all test runs.

Fig. 18 and 19 show the pressure distributions around the outer periphery of the scroll, and ahead of the inlet guide vanes, at a pressure ratio $\frac{P_{\text {to }}}{P_{\text {atm }}}=1.449$. This ratio corresponds to a mass flow rate $W_{\mathrm{Vc}}=1.875 \mathrm{lbm} / \mathrm{sec}$ which is near to that of the design point for the model turbine tests. These data were taken while performing a dummy rotor torque test and the pressure distribution is qualitatively representative of that observed for all tests with the dummy rotor. However the distribution was different if the dummy rotor was removed, indicating that the flow before the guide vanes was influenced by the aforementioned unusual conditions downstream of the guide vanes. With the dummy rotor removed it was observed that static pressures on the periphery were reduced at taps 6 and 7 , and increased at taps 9 and 10, relative to those with the dummy rotor installed. The static pressure generally decreases around the outer periphery of the scroll. This may be due to the decreasing outer radius of the scroll with angle of advance which increases the velocity there if angular momentum is conserved. The static pressure level around the outer periphery is nearly equal to that at the inlet. The static
pressure before the guide vanes around the circumference increases considerably with the angle of advance and is lower than the inlet pressure at all stations. Particularly low pressures existed at taps 1 and 2 near the inlet and at taps 8 (right and left) in the tongue of the scroll. These relatively low pressures may result from flow accelerations caused by the radial inlet pipe. The radial inlet pipe was a requirement in the original model test simulating the actual machine. It provided structural integrity for very high pressure operation. With interest directed toward performance improvement as well as performance evaluation and prediction, it would be preferable to have a tangential inlet. Conditions might also be improved by increasing the radius of curvature and/or installing a turning vane at the inlet to the scroll. The pressures measured at taps 8 (right and left) in the tongue may be influenced by the relative position of the nearest guide vane. The pressure distribution may in fact be affected by the guide vane positions, since there are seven guide vanes but eight static taps on each side. The influence of guide vane position on the pressure distribution should be checked before changes are made to the scroll area distribution. Because the pressure distribution was dependent on the configuration after the guide vanes, it would be advisable to determine the pressure distribution during turbine tests. The scroll casing has been equipped to accept total pressure probes at three of the static tap positions around the periphery.

Total pressure measurements at these stations would provide more information about the losses and the velocity distribution in the scroll. The "dynamic" pressure distribution shown in Fig. 19 is necessarily based on conditions at the scroll inlet. Measurements indicate that improvement of the pressure distribution before the guide vanes could be accomplished by increasing the cross-sectional area of the scroll in decreasing proportions with angle of advance.
6. Vortex Chamber Flow

## Installation

To obtain a better understanding of the conditions existing in the vaneless space after the inlet guide vanes, the turbine test rig was modified to carry out special tests. The existing plexiglas turbine shrouds, with contours that follow the turbine rotor profile, were replaced by flat, parallel-faced plexiglas plates which extended inward axially so that their inside faces were in the same plane as the rings supporting the inlet guide vanes. This in effect provided a flat vortex chamber in which air would be injected nearly tangentially across the width through the existing guide vanes. A cross section of this installation is shown in Fig. 20. The right-hand side plate of the vortex chamber contains an aluminum orifice plate which is recessed so that the inner face is flush with the inner face of the plexiglas plate. The orifice plate can be replaced by other plates with different orifice diameters. The plate for the present tests has a 2.50 inch orifice diameter. The so-called lefthand side plate of the vortex chamber was machined so that it could be fitted with a four-inch inside diameter aluminum pipe which would serve as a vortex tube for future experiments. This tube shown in Fig. 20 was not installed for the main part of this investigation.

The outer diameters of the abovementioned plexiglas plates of the vortex chamber are 9.50 inches, leaving a small annulus around their periphery in which the static pressure
after the guide vanes can be measured relatively free from the local influences and/or wakes of the guide vanes. Static pressure taps were also placed at different radii on the inner surfaces of the plates along a logarithmic spiral from the outer radius to the discharge radius. The logarithmic spiral represents the path of a gas particle in an incompressible, frictionless flow based on an injection angle $\alpha_{i}=79.2$ degrees at the radius $R_{i}=4.75$ inches. Initial tests were conducted with a coating of carbon-black and kerosene on the vortex chamber side plates. The observed streamlines, on the walls as shown by the carbon-black, deviate considerably from a theoretical logarithmic spiral. The car-bon-black traces show that the direction of the flow particles close to the wall have considerably reduced flow angles. This can be due to either increased radial velocity components, or decreased tangential velocity components where the latter effect is more probable. Static pressure taps were also placed at different radii along a streamline obtained from the carbon-black traces. The difference of the pressures measured were very small at the same radii along the two curves. Hence it was decided to use only the taps along the theoretical logarithmic spiral on each chamber side plate for the subsequent tests. The static pressures could be measured with an accuracy of $\pm 0.05$ inch on the 96 -inch manometer board.

A United Sensor DA-125 three-dimensional pressure probe was used to determine the flow angles and pressures across
the axial width of the vortex chamber at radii of 3.00 and 4.50 inches. The DA-125 probe is similar to the $S A-120$ probe described in section 4 but it is shorter.

The total temperature was also surveyed with the probe described in section 4 across the axial width of the vortex chamber but only at a radius of 3.00 inches. At each axial position the temperature probe was set at the same yaw angle determined by the DA-125 pressure probe. The test installation for the vortex chamber surveys is shown in Fig. 21 and 22.

## Testing and Results

To determine whether flow nonuniformities occur around the periphery of the vortex chamber after the guide vanes the pressure probe was installed at the center plane at a radius of 3.00 inches. With the probe exactly lined up with the prevailing flow directions the entire chamber end was rotated slowly while the dynamic pressure head was observed on the manometer board. Since no changes in this pressure could be observed it can be assumed that the guide vane wakes are equalized at this radius. The aforementioned procedure was also carried out at a radius of 4.50 inches. It was noticed that the dynamic head decreased about one or two percent as the probe passed through the guide vane wakes as verified by their angular separation. In these wakes the flow also became slightly more tangential. Flow angle surveys are shown in Fig. 23 for peripheral positions which can be considered to be inside and outside of the wake.

The wake position that was noticed at a radius of 4.50 inches was about 54 degrees away from the circumferential position of the guide vane trailing edge. Maximum dynamic pressure seemed to occur at a position of about 26 degrees from the wake. This position coincides with a mean streamline between guide vanes since relative positions in the guide vane flow are separated by about 51.5 degrees ( $360 \%$ ).

Fig. 23 shows that the flow angle distribution changes considerably with decreasing radius. At the 3.00 inch radius the flow angle near the center plane has increased to 90 degrees, while at the side walls the flow angle has decreased considerably from that at a radius of 4.5 inches. This increase in radial velocity components near the side walls was observed from carbon-black and kerosene test. A photograph of the vortex chamber side walls after the carbonblack and kerosene test is shown in Fig. 24.

The dynamic pressure distribution shown in Fig. 25 is nearly uniform over the axial width of the chamber but decreases at the side walls. The dynamic pressure at a radius of 3.00 inches is approximately double that at the radius of 4.5 inches.

The survey at a radius of 3.00 inches at a pressure ratio $\frac{P_{\text {to }}}{P_{\text {atm }}}=1.242$ indicated that the total pressure near the center plane of the vortex chamber was greater than that at the manifold inlet. To assure that this result was factual the pressures were measured relative to each other on the Utube water manometer. In the 4.50 inch radius surveys the
total pressure recovered in the vortex chamber was near to but not greater than the total pressure at the manifold inlet. The total pressure distribution is shown in Fig. 26. The three-dimensional pressure probe can produce measurement errors when introduced into small passages or when used near walls. Although the vortex chamber width is small (0.784 inch), the blockage effect, that can falsify static pressure readings, will be small since the flow area is the meridional cross section at the survey radius. Pitch angle and static pressure measurement may be in error near walls due to the effects of secondary flow along the probe shaft since the probe calibrations hold for deep immersion only. The pitch angles and the static pressures which are obtained from the calibration curves may therefore be inaccurate. Very near the wall the pitch angle determination is unreliable because one of the pressure sensing taps will be inside the access hole but yaw and total pressure data will still be accurate because these data do not require the use of calibration curves.

In traversing the axial width of the vortex chamber with the pressure probe it was observed that the pitch angle changed from negative near the so-called left-hand wall to positive near the so-called right-hand wall. This indicated that the flow was turned toward the wall on either side of the center plane. Although this may be due to wall effects on pitch angle determination it is more than likely that this flow pattern is an indication of the existence of axial flow
components toward the side walls and the presence of secondary flow vortices.

The temperature survey was first conducted with the inlet air at an average temperature of $121.5^{\circ} \mathrm{F}$ at pressure ratios of $\frac{P_{\text {to }}}{P_{\text {atm }}}=1.236$ and 1.484. The results of these surveys are shown in Fig. 27. A symmetrical total temperature profile existed in both runs with the highest temperatures recorded near the center plane of the chamber. The higher pressure and mass flow rate produced a larger temperature gradient across the width of the chamber. The total temperatures in the vortex chamber were below the inlet temperature at all axial positions. The inlet air temperature was considerably higher than the ambient temperature, hence temperature drops did occur because of heat transfer from the test apparatus to the surroundings. This is evidenced by the drop in total temperature of about $7^{\circ} \mathrm{F}$ from the flow measuring orifice to the manifold inlet ahead of the vortex chamber.

Another test was then made with conditions to produce nearly adiabatic conditions. The supply air was cooled as much as possible and the temperature recorded at the flowmeasuring orifice was only about $2^{\circ} \mathrm{F}$ higher than the ambient temperature in the test cell. The Kiel temperature probe at the manifold inlet was replaced by one which is identical to that used for the survey in the vortex chamber to avoid measuring inaccuracies. Before installing this probe its output was compared in still air with the one used for the survey and no difference could be observed. Surveys were
then made at a radius of 3.00 inches at pressure ratios of $\frac{P_{\text {to }}}{P_{\text {atm }}}=1.236$ and 1.484. Since the inlet temperature was increasing slightly during the test time, the difference between the temperature recovered at axial positions in the vortex chamber and the total temperature at the inlet is shown in Fig. 28. A symmetrical axial temperature profile was obtained again but the temperature difference is positive near the center and negative near the side walls. Since it can be assumed that the instrumentation is reliable, it appears that the total temperature recovered near the center plane of the vortex chamber is higher than that at the inlet. The possibility of heat being transfered from the surroundings to the flow must be ruled out since the contact areas where high velocities exist are small. It seems feasible therefore that energy separation is taking place in the flow and that the flow at the side walls gives up energy to that in the center. This result supports the measured fact of a higher total pressure near the center plane of the vortex chamber than at the manifold inlet, even with an increase in entropy due to frictional effects.

Limited testing was conducted with a 42 -inch by fourinch inside diameter vortex tube attached to the left end of the vortex chamber. A conical valve was inserted in the tube end to divide the flow discharge between the annulus, thus formed, and the orifice on the opposite end. Total temperature separations of nearly $30^{\circ} \mathrm{F}$ were sensed at a pressure ratio $\frac{\mathrm{P}_{\text {to }}}{\mathrm{P}_{\text {atm }}}=1.5$. Instrumentation was employed to
to demonstrate the effect in an approximate manner; however, the installation could readily accept instrumentation with very little disturbance to the flow because of its size and the large flow capacity of the facility.

## Ranque-Hilsch Effect

Many studies have been conducted on the behavior of vortex flow in cylinders. Much of this interest was concerned with the Ranque-Hilsch tube. In such a device compressed gas is injected tangentially at the circumference near one end of a tube. This results in a vortex core near the axis which is at a lower total temperature than that of the inlet gas, whereas the gas in the annulus surrounding the core is at higher total temperature. Different configurations have been utilized to separate the hot and cold flows. Usually the cold stream is extracted through an orifice on the injection end while the hot stream discharges from an annular region at a larger radius on the opposite end of the tube. However, a counterflow configuration such as this is not essential in producing the separation. The process is referred to as the Ranque-Hilsch effect, total temperature separation, or energy separation. Harnett and Eckert [9] in experiments they conducted with a vortex tube found that the greatest energy separation took place in the tube cross section near the tangential nozzles. Work by Savino and Ragsdale $[10]$ indicates that considerable energy separation can take place in a vortex contained between two flat plates without an attached tube. In their experiment, the flow
emerged from an opening at the center with a diameter on the order of the plate spacing; and most of the energy separation took place near the opening. In the present investigation higher total temperatures and pressures than those at the inlet were found to occur at the smaller survey radius of 3.00 inches.

Deissler and Perlmutter [11] concluded from an analytical investigation that the most important factor affecting the total temperature of a fluid element in a compressible vortex is the turbulent shear work done on or by the element. Although no conclusive evidence exists as to the exact mechanism causing energy separation, there seems to be general agreement that it is brought about by the so-called turbulent or eddy viscosity. A parameter on which energy separation seems to be dependent is a turbulent radial flow Reynolds number which is expressed either with the eddy viscosity only, or together with the molecular viscosity. The turbulence of the flow causes the effective Reynolds number to remain low even when the Reynolds number based on molecular viscosity is high. This occurs because the turbulent viscosity may be several orders of magnitude higher than the molecular viscosity. In some analyses the molecular viscosity is neglected. It appears, however, that there is an optimum turbulence for separation since diffusion will counteract the separation caused by increased viscosity if turbulence is increased indefinitely.

It appears also that energy separation depends on the deviation of the tangential velocity profile from that for an inviscid flow. Since a free vortex flow requires the impossible condition of infinite velocity at the axis, such deviations will always occur in flows of real fluids. In section 4 the survey of the turbine rotor discharge section, with the rotor removed, showed that the velocity decreases with decreasing radius. The velocity profile is more nearly that of a forced vortex or rotating solid than the one which would occur in a free vortex flow. The discharge flow of the vortex chamber was not surveyed but results of the axial traverse at a radius of 3.00 inches indicate that the flow near the mid-plane deviates considerably from inviscid flow since there was no radial velocity component. Some investigators feel that temperature separation will only appear in regions where an axial variation of the mass flow can be . established such as would occur near the discharge area.

## Vortex Chamber One-Dimensional Analysis

The flow in a vortex chamber with parallel side walls perpendicular to the axis of symmetry can be considered to be axisymmetric. If the flow is uniformly injected at the radius $R_{i}$ with a known Mach number $M_{i}$ and flow angle $\alpha_{i}$, a one-dimensional analysis can be made to determine the Mach number $M$ and flow angle $\alpha$, at an arbitrary radius $R$ less than $R_{i}$. Such an analysis has been carried out by Vavra [12]. The flow is considered to be compressible and steady. It is assumed that frictional losses are negligible
and the process is adiabatic. The analysis assumes no velocity components or velocity gradients in the axial direction. If the width of the chamber is $b$, continuity is satisfied at an arbitrary radius $R$ by

$$
\rho 2 \pi R b V_{m}=\rho_{i} 2 \pi R_{i} b V_{m i}
$$

or since $V_{m}=V_{\cos \alpha}$ and $V_{m i}=V_{i} \cos \alpha_{i}$

$$
\rho R V \cos \alpha=\rho_{i} R_{i} V_{i} \cos \alpha_{i}
$$

By introducing the acoustic velocities, there is

$$
\rho R M \cos \alpha=\rho_{i} R_{i} M_{i} \cos \alpha_{i}\left(\frac{a_{i}}{a}\right)(6-1)
$$

With the flow being frictionless, no moment is exerted by the walls on the flow, and angular momentum is conserved. Then at an arbitrary radius $R$

$$
R V_{u}=R_{i} V_{u i}
$$

with

$$
V_{u}=V \sin \alpha \text { and } V_{u i}=V \sin \alpha_{i}
$$

there are

$$
R V \sin \alpha=R_{i} V_{i} \sin \alpha_{i}
$$

and

$$
\begin{equation*}
R M \sin \alpha=R_{i} M_{i} \sin \alpha_{i}\left(\frac{a_{i}}{a}\right) \tag{6-2}
\end{equation*}
$$

From equations (6-1) and (6-2)

$$
\begin{equation*}
\tan \alpha=\tan \alpha_{i}\left(\frac{\rho}{\rho_{i}}\right) \tag{6-3}
\end{equation*}
$$

Since the flow is assumed to be adiabatic, total energy is conserved and

$$
c_{p} T+\frac{V^{2}}{2 g J}=c_{p} T_{i}+\frac{V_{i}^{2}}{2 g J}
$$

With

$$
c_{p}=R_{g} \frac{\gamma}{\gamma-1}
$$

and the acoustic velocities,

$$
\begin{equation*}
1+\frac{\gamma-1}{2} M^{2}=\frac{a_{i}^{2}}{a^{2}}\left(1+\frac{\gamma-1}{2} M_{i}^{2}\right) \tag{6-4}
\end{equation*}
$$

Since the flow is assumed to be isentropic

$$
\frac{a_{i}^{2}}{a^{2}}=\frac{T_{i}}{T}=\left(\frac{\rho_{i}}{\rho}\right)^{\gamma-1}=\left(\frac{p_{i}}{p}\right)^{\frac{\gamma-1}{\gamma}}
$$

$$
\begin{aligned}
& \text { Then, with pressure ratio, } p / p_{i} \\
& \qquad M=\left\{\frac{2}{\gamma-1}\left[\left(\frac{P_{i}}{P}\right)^{\frac{\gamma-1}{\gamma}}\left(1+\frac{\gamma-1}{2} M_{i}^{2}\right)-1\right]\right\}^{\frac{1}{2}}
\end{aligned}
$$

and

$$
\begin{equation*}
\tan \alpha=\tan \alpha_{i}\left(\frac{p}{p_{i}}\right)^{\frac{1}{\gamma}} \tag{6-6}
\end{equation*}
$$

Mach number $M$ and flow angle $\alpha$ are then known as a function of inlet conditions and pressure ratio. The radius can be determined from equation ( $6-2$ ) by replacing the ratio of the acoustic velocities by the pressure ratio, giving

$$
\begin{equation*}
R=R_{i} \frac{M_{i} \sin \alpha_{i}}{M \sin \alpha}\left(\frac{p_{i}}{P}\right)^{\frac{\gamma-1}{2 \gamma}} \tag{6-7}
\end{equation*}
$$

To apply this analysis to the results obtained in tests it is necessary to know $M_{i}$ and $\alpha_{i}$ at some radius $R_{i}$ in the vortex chamber. Pressures and temperatures were not available in the same test run, hence the Mach number $M_{i}$ is unknown. By extrapolating data from the dummy rotor torque tests there are $M_{i}=.175$ and $\alpha_{i}=79.2^{\circ}$ at $R_{i}=4.75$ inches, at $\frac{P_{\text {to }}}{P_{1}}=\frac{P_{\text {to }}}{P_{i}}=1.024$. The theoretical pressure distribution obtained with these conditions is shown with a
broken curve in Fig. 29. Although the form of the curve compares well with the actual static pressure distribution it does not coincide with it. However, since the theoretical pressure distribution depends greatly on the value of $M_{i}$ an exact comparison is difficult.

Fig. 30 shows the results of the theoretical analysis for $M_{i}=0.175$. It is of interest to note that supersonic flow could be obtained without converging-diverging flow areas at a radius ratio $R / R_{1}$ less than about $18 \%$ but the flow does not choke until a ratio of $R / R_{1}$ of about $15 \%$ is reached. Choking can occur only if the Mach number of the radial velocity component is unity whereas the Mach number of the actual velocity is greater than unity. An oblique or normal shock can occur in the flow only if the component of velocity normal to the shock is supersonic. From consideration of the axial symmetry of the flow in the vortex chamber, this would require that the shock be a circular cylinder. Then the flow could not be choked with a subsonic radial velocity component even though the velocity is supersonic because there are large peripheral components.

These interesting aspects of the vortex chamber have not been realized in the subject investigation. To acquire survey data and to remain within the capability of the instrumentation as it was initially installed, the maximum pressure ratio at which tests were made was $\frac{\mathrm{P}_{\text {to }}}{\mathrm{P}_{\text {atm }}}=1.553$. This is well below that which would theoretically produce supersonic flow; however, it appears from an extension of
the test data that the required theoretical pressure ratio is within the capability of the compressor. Also since the flow becomes supersonic at a larger radius ratio if the injection Mach number is higher, the existing vortex chamber geometry may meet the requirements.

The influence of viscosity in the vortex chamber flow, however, could prevent supersonic flow conditions. The flow paths, especially near the center plane of the chamber, are long, and viscous effects dominate. That these effects are large has been demonstrated by the departure of the flow angle of the real flow from the theoretical one. The increased radial velocity components near the side walls must result from wall friction but it appears that they may be initiated by secondary flow in the curved channels between the inlet guide vanes before the flow enters the chamber.

The considerable change in flow angle across the axial width of the vortex chamber is of interest to the turbine designer. Within the limitations imposed by structural and manufacturing requirements the rotor tip could be designed to accept the flow as it actually exists and it seems possible to reduce the losses with such a procedure.

The flow in a vortex chamber with the geometry investigated is truly three-dimensional. The flow develops quickly and there appears little possibility of separating a "boundary layer" on the side walls from a potential flow near the center plane. Even in a chamber of this geometry, where the axial width is small compared to the diameter,
there appears to be evidence of axial velocity components which should be considered in an analysis. Knowledge about turbulent flows and turbulent boundary layers in general is limited. Laws predicting separation for even two-dimensional boundary layer flows are inadequate. In the vortex chamber the flow is complicated by the existence of a pressure gradient and the three-dimensional aspects, including the fact that there is more than one large velocity component. Complete analysis of the flow is a problem of great magnitude.



Fig. 3 Four-inch Pipe with Remote Controlled Valve and Flow-measuring Orifice


Fin, 4 Heise Cage and frown Potemitometer



Fic. Dram ko ir rest Installation

Fig. 8
VELOCITY COEFFICIENT
vs
PRESSURE RATIO

0.94
0.93
0.92
$\phi$
0.91


0.93
0.90
$\nabla$


Fig. 9
VELOCITY VEOEFFICIENT
PRESSURE RATIO


Fig. 10
ABSOLUTE ROTOR INLET ANGLE ve.
PRESSURE RATIO


mLSSSURE PROEE HEAD TEMPERATURE PROEE HEAD
FNESSURE AMO TEMPERATUNE SUNVEY PROEE HEADS


Fig. 13









Fig. 21 Vortex Chamber Tnstallation










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## APPENDIX A

Program Scroll
Program SCROLL computes the losses in the scroll and guide vanes, and the absolute rotor inlet flow angle from torque, flow rate, and pressure data obtained in dummy rotor tests. The program is essentially that used by Riley [2] with several changes. The program can process any number of runs, with a maximum of ten sets of data per run. A block diagram of the program is shown in Fig. A1 and a program listing is given in Table A1.

A description of the main program and the subroutines is given below. The program initially reads the number of runs (NRUNS) which is used as an upper limit for the first DO loop. Within the DO loop, the number of sets of data (NSETS) and the input data for the run are read into the program. The input data is then printed out and the processing of each set of data is started using a DO loop with index $K$ varying from 1 to NSET.

The value of the specific gravity of mercury $G_{h g}$ at room temperature $t_{r m}$ is determined by

$$
\begin{equation*}
G_{H g}=13.638-1.354\left(10^{-3}\right) \mathrm{trm} \tag{A-1}
\end{equation*}
$$

for temperatures between $0^{\circ} \mathrm{F}$ and $150^{\circ} \mathrm{F}$. The factor for converting in. Hg to $\mathrm{lb} / \mathrm{ft}^{2}$ is

$$
\begin{equation*}
c_{f}=69.892 \frac{G_{H g}}{13.59} \tag{A-2}
\end{equation*}
$$

The value of the specific gravity of water, $G_{H 2 O}$, at room temperature is

$$
G_{\mathrm{H}_{2} \mathrm{O}}=1.00013+7.8\left(10^{-5}\right) t_{\mathrm{rm}}-1.4\left(10^{-6}\right) t_{\mathrm{rm}}^{2}(\mathrm{~A}-3)
$$

The specific gravity relations were obtained from tabulated data of $[4]$.

After the data for one run have been processed by the four subroutines, discussed in sections A1 through A4, the data are printed out and those for the next run are read into the program.

A1. Subroutine TEMP
Subroutine TEMP calculates the total temperature ahead of the flow measuring orifice, and at the manifold inlet, from chromel-alumel thermocouple readings. Using the measused voltage MV in milli-volts and the cold junction temperature $t_{c j}$ the relations for the evaluation of the temperature in TEMP are

$$
\begin{equation*}
t=t_{c j}+44.41 \mathrm{MV}+0.2185 \mathrm{MV}^{2} \tag{A-4}
\end{equation*}
$$

for $t \leq 100^{\circ} \mathrm{F}$

$$
\begin{equation*}
t=t_{c j}+45.24 M V-0.3295 M V^{2} \tag{A-5}
\end{equation*}
$$

for $100^{\circ} \mathrm{F}<t \leq 200^{\circ} \mathrm{F}$
Equations (A-4) and (A-5) were obtained from data tabulated in [6].

A2. Subroutine FLOW
Subroutine FLOW calculates the flow rate using only the pressures obtained with the vena-contracta taps of the orifice since this data gives a more accurate flow rate than flange tap data. ${ }^{1}$ The flow rate is measured with a sharp edge orifice of 2.800 inch diameter which is installed in a pipe of 4.026 inch I.D.

The relation for the flow rate is ${ }^{2}$

$$
\begin{equation*}
\dot{W}_{V C}=c \propto Y_{1} F_{r} \sqrt{\frac{P_{i V c} \Delta h_{1 V C}}{T_{4}}} \tag{A-6}
\end{equation*}
$$

where:

$$
\begin{aligned}
& \text { C - factor dependent on orifice diameter, } \\
& \text { type of pressure taps, and dimensional units } \\
& \alpha \text { - area multiplier to account for the thermal } \\
& \text { expansion of the orifice } \\
& Y_{1} \text { - expansion factor to account for compress- } \\
& \text { ability effects } \\
& F_{r} \text { - Reynolds number correction factor } \\
& P_{\text {Vc }} \text { - absolute pressure at upstream tap } \\
& h_{1 v c}-\text { pressure differential across orifice } \\
& \mathrm{T}_{4} \text { - temperature ahead of the orifice }
\end{aligned}
$$

For vena-contracta $\operatorname{taps}^{3} \mathrm{C}=0.9057$
For a steel orifice ${ }^{4}$ :

$$
\begin{align*}
& \text { steel orifice }:  \tag{A-7}\\
& \alpha=1+\left(T_{4}-530\right)\left(10^{-3}\right)
\end{align*}
$$

1Vavra, M. H., Results of Turbine Air Testing Program, Phase II, Report ALGER No. 29, for Aerojet General Corporation (1965), p. 219 .

$$
\begin{aligned}
& 2 \text { Ibid., p. } 220 . \\
& 3 \text { Ibid., p. } 221 . \\
& 4 \text { Ibid., p. } 220 .
\end{aligned}
$$

and

$$
\begin{align*}
& Y_{1}=1-0.351 \frac{\Delta h_{1 v c}}{P_{1 v c}}  \tag{A-8}\\
& F_{r}=1+\frac{0.00114}{X} \tag{A-9}
\end{align*}
$$

For an orifice diameter of 2.800 inches $^{5}$

$$
\begin{equation*}
X=0.812 \frac{W_{v c}}{Z} \tag{A-10}
\end{equation*}
$$

where for air between $50^{\circ} \mathrm{F}$ and $300^{\circ} \mathrm{F}$

$$
\begin{equation*}
Z=1.9+2.4\left(T_{4}-560\right)\left(10^{-3}\right) \tag{A-11}
\end{equation*}
$$

Since $F_{r}$ is nearly unity, the flow rate ${ }_{W}{ }_{V C}$ * is determined first for $\mathrm{F}_{\mathrm{r}}=1$. This flow rate is used to determine $X$ by using Eq. ( $A-10$ ), then the actual flow rate is taken as

$$
\begin{equation*}
\dot{W}_{V C}=F_{r} \dot{W}_{V C} \tag{A-12}
\end{equation*}
$$

without further iteration.
$P_{1 v c}$ and $\Delta h_{1 v c}$ are converted to the proper units from their respective measured values by the relations
$P_{1 v c}=\left(P_{1 v c}{ }^{\prime}-t a r e+2.54 P_{a t_{m}}\right) \frac{G_{H g}}{13.59}$
and
$\Delta h_{1 v c}=\left(\Delta h_{1 v c}{ }^{\prime}-\right.$ tare $) \frac{G H g}{13.59}$
A3. Subroutine PRESS
Subroutine PRESS determines the static pressure ahead of the dummy rotor, and the ratio of the total pressure at Ibid., p. 220.
the manifold inlet and the static pressure ahead of the dummy rotor.

The static pressure at the rotor inlet $p_{1}$ is obtained from the average of the measured rotor inlet pressure ( $\mathrm{atm}^{-h_{1}}$ ), where $h_{1}$ is the average of the pressure readings, and $h_{a t m}$ is the reference pressure. Then

$$
\begin{equation*}
p_{1}=\frac{\left(P_{a t m}\right) G_{H g}+\left(h_{a t_{m}}-h_{1}\right) G_{H_{2} O}}{13.59} \tag{A-15}
\end{equation*}
$$

From the measured static pressure $\mathrm{p}_{5}$, at the manifold inlet the absolute static pressure $p_{0}$ at the inlet is

$$
\begin{equation*}
P_{0}=\left(\frac{P_{5}{ }^{\prime}-\text { tare }^{2 .}}{2.54}+P_{\text {atm }}\right) \frac{G_{H g}}{13.59 .} \tag{A-16}
\end{equation*}
$$

An iteration process is used to determine an average value of the total pressure at the turbine inlet. Three relations are used in the iteration, namely, the gas law,
the continuity equation,

$$
\begin{equation*}
\rho_{0}=C_{f} \frac{\rho_{0}}{R_{g} T_{0}} \tag{A-17}
\end{equation*}
$$

$$
\begin{equation*}
V_{0}=\frac{\dot{W}_{v c}}{\rho_{0} A_{5}} \tag{A-18}
\end{equation*}
$$

and the energy equation,

$$
\begin{equation*}
T_{0}=T_{0}-\frac{10}{20} \tag{A-19}
\end{equation*}
$$

where $A_{5}$ is the area of the five-inch pipe, and $c_{p}$ is the specific heat of air at $T_{\text {to }}$. The variation of the specific heat at constant pressure $c_{p}$ with temperature obtained from data tabulated in [5], is

$$
\begin{equation*}
c_{p}=0.23943+3.4\left(10^{-6}\right) t+2\left(10^{-8}\right) t^{2} \tag{A-20}
\end{equation*}
$$

Using $T_{\text {to }}$ for the first approximation of $\rho_{0}$, the iteration continues until a difference of $0.01^{\circ}$, or less, exists between any two successive values of $T_{0}$. Then the average total pressure $P_{\text {to }}$ is

A4. Subroutine PSI
Subroutine PSI determines the inlet velocity coefficient $\phi$ and the absolute rotor inlet flow angle $\alpha_{1}$.

From the theorem of angular momentum ${ }^{1}$, for a steady flow that does not have a whirl component at the dummy rotor discharge $\left(V_{u 2}=0\right)$ the moment $M$ exerted on the dummy notor with radius 4.75 inches is

$$
\begin{equation*}
M=\dot{W}_{v c} \frac{4.75 V_{u s}}{9} \tag{A-22}
\end{equation*}
$$

The moment $M$ equals the product of the scale reading $F$ and the length of the moment (12 inches). Thus the peripheral component of the absolute rotor inlet velocity $V_{1}$ is from Eq. (A-22)

$$
\begin{equation*}
V_{u 1}=\frac{12 F_{q}}{4.75 W_{v c}} \tag{A-23}
\end{equation*}
$$

The velocity coefficient $\boldsymbol{\phi}$ is determined by an iteration using the rotor inlet velocity $V_{1}$. The first approximation to $\mathrm{V}_{1}$ is obtained by assuming an isentropic expansion from the manifold inlet total pressure $P_{\text {to }}$ to the rotor inlet pressure $p_{1}$; that is, the velocity coefficient $\phi$ is
${ }^{1}$ Vara, M. H. Aero-Thermodynamics and Flow in Turbomachines (John Wiley and Sons, 1960), p. 98.
set equal to unity, and

$$
\begin{equation*}
V_{1}=\sqrt{2 g J c_{p} T_{t o}\left[1-\left(\frac{p_{1}}{P_{t_{0}}}\right)^{\frac{\gamma-1}{\gamma}}\right]} \tag{A-24}
\end{equation*}
$$

The variation with temperature of the ratio of specific heats $\gamma$ obtained from data tabulated in [5] is

$$
\begin{equation*}
\gamma=1.4018-2\left(10^{-5}\right) t \tag{A-25}
\end{equation*}
$$

The meridional component of $V_{1}$ is

$$
\begin{equation*}
V_{m 1}=\sqrt{V_{1}^{2}-V_{u 1}^{2}} \tag{A-26}
\end{equation*}
$$

Using the continuity equation, the density $\rho_{1}$ is

$$
\begin{equation*}
\rho_{1}=\frac{\dot{W}_{V c}}{A_{1} V_{m 1}} \tag{A-27}
\end{equation*}
$$

where the meridional cross-sectional area $A_{1}$ is

$$
\begin{equation*}
A_{1}=\frac{\pi(9.5) B_{1}}{144} \tag{A-28}
\end{equation*}
$$

$B_{1}$ is the distance between the shrouds at the dummy rotor tip radius of 4.75 inches and depends on the axial clearance used for the test. Using the gas law, the static inlet femperature $T_{1}$ is

$$
\begin{equation*}
T_{1}=C_{f} \frac{p_{1}}{\rho_{1} R_{g}} \tag{A-29}
\end{equation*}
$$

The second approximation of $V_{1}$ is

$$
\begin{equation*}
V_{1}=\sqrt{2 g J_{c_{p}}\left(T_{t_{0}}-T_{1}\right)} \tag{A-30}
\end{equation*}
$$

By reducing $\phi$ in increments of 0.0001 until the two approximations for $V_{1}$ agree within $1.0 \mathrm{ft} / \mathrm{sec}$. the actual value of $\phi$ is obtained.

The absolute rotor inlet flow angle $\boldsymbol{\alpha}_{1}$ is then

$$
\begin{equation*}
\alpha_{1}=\tan ^{-1} \frac{V_{u_{1}}}{V_{m 1}} \tag{A-31}
\end{equation*}
$$

From the velocity $V_{1}$ and the static temperature $T_{1}$, the Mach number $M_{1}$ at the rotor inlet is

$$
\begin{equation*}
M_{1}=\frac{V_{1}}{\sqrt{\gamma R_{g} g T_{1}}} \tag{A-32}
\end{equation*}
$$



Fig. Al
Block Diagram of Program SCROLL
Table Al Program SCROLL Listing

$\operatorname{PRP}(K)=P R$
$\operatorname{PHIP}(K)=\operatorname{PHI}$
$\operatorname{ALPH}(K)=A L P I$
$\operatorname{VEL}(K)=V 1$
$\operatorname{TS}(K)=T 1$
$\operatorname{ACH}(K)=A C H 1$
$\operatorname{PS}(K)=P 1$
$\operatorname{CONTINUE}$
$\operatorname{PRINT} 22$
$\operatorname{PRINT} 23,(K, \operatorname{PRP}(K), W V C P(K), P H$
 Table Al (Cont.)
PRINT $23,(K, \operatorname{PRP}(K)$, WVCP $(K), P H I P(K), A L P H(K)$,
IVEL $(K), T S(K), A C H(K), P S(K), K=1, N P T S)$
99 CONTINUE
PRINT 25
10 FORMAT 14$)$
11 FORMAT $10 F 7.2)$
12 FORMAT $3 F 7.2$, F6.3)
20 FORMAT $1 H 1 / 12 \times 14 H P R O G R A M ~ S C R O L L 50 \times 13 H R . L . B O S H$


$100 T=T C J+44.41 * V+.2185 * V * * 2$


Table Al (Cont.)
$T 0=T 5-(V 0 * * 2) /(2 \cdot * 32 \cdot 174 * 778 \cdot 16 * C P)$
$T 0=T 5-(V 0 * * 2) /(2 \cdot * 32 \cdot 174 * 778 \cdot 16 *(P)$
OTT $\mathrm{T}=\mathrm{T} T-\mathrm{TO}$
IF (ABSF (DTT)-.01) $101,101,100$
PTO $=$ PS5 +RHO*(VO**2)/(2.*32.174*CF1)
PR $=$ PTO/P1
RETURN
END
-1
-
-

## SUBROUT INE ALPHPSI (SR) COMMON GHG,GWR,CFI,TCJ,

COMMON GHG, GWR,CFI,TCJ,TARE,STARE,T4,T5,WVC,PR,Pl,PHI,ALPI,PAT,

$v$



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TABLE A

••••••••

| FICP | RADIAL | TURBINE |
| :---: | :---: | :---: |
| DATA | Run 3 |  |
| H1 | SR | TRM |
| 0.80 | 1.85 | 70.00 |





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| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
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| 02.02 | 8L．${ }^{\text {ci }}$ | ¢101t | 06.99 | 86.62 | 280． 2 | $02 \cdot 09$ | 60：T |  |
| $00^{\circ} 02$ | 91．${ }^{\text {ct }}$ | 号でで | － 09.89 | 86.62 | 28． 2 | 20：¢ ¢ | 20：61 |  |
| 08.69 | $00^{\text {－} 11}$ | 28．とう | S1．8S | $86 \cdot 6$ 2 | GL・て乙 | 6ヶ・的 | $6 \square^{-51}$ |  |
| 07.69 | ¢ 7508 | $18.2 \square$ | 2I．${ }^{\text {¢ }}$ | $86 \cdot 62$ | を业： | 50.62 | EG：It |  |
| 00：69 | $29 \cdot \frac{}{2}$ 19 | 280． LC | S L6． 25 | 886.62 |  | 10.61 08.8 | \％ 89 ${ }^{\circ} \mathrm{L}$ |  |
| W女1 | ys | IH | W1 $\mathrm{VH}^{\text {d }}$ | WIVd | dSd | J And | Jndo |  |
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PROGRAM SCROLL



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## Scroll Area Distribution

For scrolls with circular cross sections, Eckert [13] has derived an approximate equation for the radius of the cross section as a function of the angle of advance assuming frictionless incompressible flow. If the nomenclature is changed for application to a turbine scroll

$$
r=\frac{360^{\circ}-\delta}{c}+\sqrt{\frac{2 r_{1}}{c}\left(360^{\circ}-\delta\right)}(f t .)(B-1)
$$

where $r_{1}$ is the inside radius of the scroll, $\delta$ the circumferential angle of advance measured from the inlet pipe centerline, and $c$ a constant defined by

$$
\begin{equation*}
c \equiv \frac{720 \pi V_{\delta 1} r_{1}}{V^{-}}\left(\frac{\text { deg. }}{f t}\right) \tag{B-2}
\end{equation*}
$$

${ }^{\mathrm{V}} \mathrm{S}_{1}$ is the tangential velocity component at the inside radias of the scroll, and $\sqrt{ } \mathcal{F}$ is the volume flow rate at the inlet. Since the radius at the inlet of the existing scroll is $r=0.2083$ feet and the radius $r_{1}=0.5779$ feet, equation (B-1) can be solved for $c$ at $\delta=0$. The resulting $c=12,800 \frac{\text { deg. }}{\text { ft. }}$ can then be used to determine $r$, and hence the cross-sectional areas of the scroll at various $\delta$. The calculated cross-sectional area of the scroll can be corrected for frictional losses to maintain a constant pressure distribution around the periphery. The ratio of corrected to uncorrected radius is

$$
\begin{equation*}
\frac{r_{f}}{r} \equiv \frac{1}{\sqrt[4]{1-\Delta q}} \tag{B-3}
\end{equation*}
$$

where $\Delta q$ is a non-dimensional head loss, defined below, and determined from the empirical relation
$\Delta q \equiv \frac{2 g \Delta H}{V_{\delta 1}^{2}}=0.0233 f c r_{1} \ln \left[\frac{\sqrt{2 r_{1} c}+3 \sqrt{360^{\circ}}}{\sqrt{2 r_{1} c}+3 \sqrt{360^{\circ}-\delta}}\right](B-4)$
The skin friction coefficient, $f$, is estimated to be 0.0025 . The corrected area is

$$
\begin{equation*}
A_{\delta f}=\frac{A_{\delta}}{\sqrt{1-\Delta q}} \quad\left(i .^{2} .\right) \tag{B-5}
\end{equation*}
$$

The calculated areas are tabulated in Table B1 and the area distribution is shown graphically in Fig. 17.

TABLE BI

| $\delta($ deg. $)$ | $A_{\delta}\left(i n_{1}^{2}\right)$ | $\Delta q$ | $A_{\delta f}\left(i n^{2}\right)$ |
| :---: | :---: | :---: | :---: |
| 0 | 19.646 | 0 | 19.646 |
| 30 | 17.802 | 0.0059 | 17.854 |
| 75 | 15.089 | 0.0152 | 15.205 |
| 120 | 12.459 | 0.0259 | 12.624 |
| 165 | 9.894 | 0.0379 | 10.087 |
| 180 | 9.062 | 0.0422 | 9.260 |
| 210 | 7.414 | 0.0514 | 7.612 |
| 255 | 5.036 | 0.0684 | 5.218 |
| 300 | 2.764 | 0.0902 | 2.897 |
| 330 | 1.338 | 0.1107 | 1.419 |
| 360 | 0 | 0.1654 | 0 |

## APPENDIX C

Data Reduction
Data reduction to obtain scroll and guide vane performance from torque tests was carried out by program SCROLL. Program SCROLL is described in detail in Appendix A. The reduction of data for the other investigations is described below.

C1. Flow Rate Calculation
Flow rates were calculated using the same relations and procedure as described in section A1 of Appendix A for digital computer calculation.

C2. Temperature
The recorded output in milli-volts of the chromedalumel and iron-constantan thermocouples was directly reduce to temperature using data tabulated in $[6]$ and interpolating for fractions of degrees as necessary.

C3. Pressures
The specific gravities of mercury $G_{H g}$ and water $G_{H_{2} O}$
used in the manometers were calculated for control room temperature $t_{r m}$ using equations $(A-1)$ and $(A-2)$ which were obtained from data tabulated in $[4]$.

The absolute static pressure at the manifold inlet, determined from a micromanometer reading, is

$$
\begin{equation*}
p_{0}=\left(p_{5}^{\prime}-\operatorname{tare}+2.5 .4 P_{\mathrm{atm}}\right) \frac{G_{H_{q}}}{13.59} \tag{C-1}
\end{equation*}
$$

The absolute total pressure at the manifold inlet is

$$
\begin{equation*}
P_{t_{0}}=p_{0}+2.54\left(h_{\text {ref }}-h_{t_{0}}\right) \frac{G_{H_{2} \mathrm{O}}}{13.59} \tag{C-2}
\end{equation*}
$$

when the static pressure at the manifold inlet is the reference pressure. Pressures measured on the 96-inch water manometer board are

$$
\begin{equation*}
p=p_{0}+2.54\left(h_{r e f}-h\right) \frac{G_{\mathrm{H} 2 \mathrm{O}}}{13.59} \tag{C-3}
\end{equation*}
$$

Pressure ratios and differences are calculated as required.

C4. Pressure Probe Survey Data
Fig. C1 and C2 are calibration curves for the threedimensional probes used in surveys at the rotor discharge section, and vortex chamber, respectively. Although the curves differ from each other, the procedure used to determine actual pressures and flow angles from recorded data is the same. The pitch angle $\theta$ is determined from the calibration curve by entering with the pitch angle pressure coefficient $\frac{\mathrm{p}_{4}-\mathrm{p}_{5}}{\mathrm{P}_{1}-\mathrm{p}_{2}}$ that is obtained from measured data. The subscripts refer to the tap on the probe at which the pressure is measured. The positions of these taps are shown in Fig. 11. With the pitch angle $\theta$, the velocity pressure coefficient $\frac{\mathrm{P}_{t}-\mathrm{p}_{S}}{\mathrm{P}_{1}-\mathrm{p}_{2}}$ is established from the calibration curve for the $M_{r}$ range where $M_{r}$ is defined as $\frac{P_{1}-p_{2}}{P_{1} \text { (absolute). }}$ Since $\left(P_{1}-p_{2}\right)$ is measured, the corrected dynamic pressure $\left(P_{t}-p_{s}\right)$ is determined. The total pressure coefficient
$\frac{P_{1}-P_{t}}{P_{t}-p_{s}}$ is very nearly zero in the range of moderate pitch angle, hence the measured total pressure requires no correction. The total pressure was measured against atmospheric pressure, hence total and static absolute pressure, as well as pressure ratios, are obtained by using the prevailing barometer reading.


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This investigation was conducted to determine the losses in the scroll and inlet guide vanes of a dual discharge, radialinflow turbine. Difficulties are encountered in such tests because the air is discharged from the guide vanes with a large whirl component into the cavity normally occupied by the turbine rotor. Connected with such flows are phenomena such as choking and energy separation. This led to the investigation of the flow in a.vortex chamber where the vortex is driven by the inlet guide vanes of the radial turbine.

The air tests were conducted at the Turbo-Propulsion Laboratory of the Naval Postgraduate School, Monterey, California.

[14. $\quad$| whirling flow |
| :--- |
| vortex chamber |
| radial turbine |

